

# *Hello future Algebra I Students! Here is your Algebra I Summer, 2025 Packet!*

- If you want, you can show work and answers on a separate paper.
- Answers to all of the problems are on pages 17 and 18. It is important to have immediate feedback so you can make sure you are doing the problems correctly. Of course, you can just *copy* the answers, but hopefully you are far enough on your educational journey to realize that is not helpful at all!
- There will be an assessment over this material sometime within the first few weeks of the 2023 – 24 school year.
- ***You do not have to complete this packet all at once!*** Pace yourself over the summer. Here is a suggestion: divide the problems and do a few from each section several times (or “sessions”) over the summer. For example:

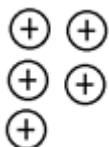
	<b><i>Session 1</i></b> (perhaps mid-June?)	<b><i>Session 2</i></b> (perhaps late June?)	<b><i>Session 3</i></b> (perhaps mid-July?)	<b><i>Session 4</i></b> (perhaps late July?)	<b><i>Session 5</i></b> (perhaps early August?)
Problem Set I (page 3)	1 – 20	21 – 40	41 – 60	61 – 80	50 / 20 / 10: What Every Algebra Student Should Know (pages 15 – 16)
Problem Set II (pages 5 - 6)	81 – 86	87 – 92	93 – 98	99 – 106	
Problem Set III (pages 7 - 9)	107 – 114	115 – 122	123 – 130	131 – 138	
Problem Set IV (page 10)	139 – 140	141 – 142	143 – 144	145 – 147	
Problem Set V (page 11)	148 – 149	150 – 151	152 – 153	154 – 155	
Problem Set VI (pages 12 - 14)	156 – 159	160 – 163	164 – 167	168 - 170	

- **And finally...** if you don’t understand perfectly how to do a problem or a certain set of problems, you can refer to the explanations and examples at the beginning of each problem set. If you still have some problems, we can address them during the first few days of school.

## Combining (Adding and Subtracting) Integers: Explanations and Examples

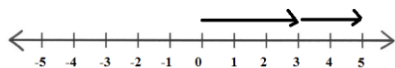
### Signs are the same! ADD

$$3 + 2 = 5$$

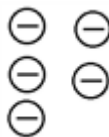


More positives

→ answer is positive

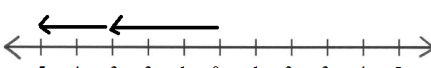


$$-3 - 2 = -5$$



More negatives

→ answer is negative



### Examples

$$3 + 6 = 9$$

$$-3 + (-6) = -9$$

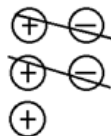
$$-3 - 6 = -9$$

$$-4 + (-5) + (-6) = -15$$

$$-4 - 5 - 6 = -15$$

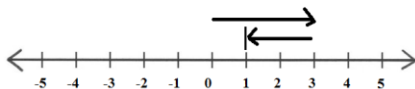
### Signs are different! SUBTRACT

$$3 - 2 = 1$$

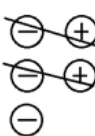


More positives

→ answer is positive



$$-3 + 2 = -1$$



More negatives

→ answer is negative



### Examples

$$10 - 4 = 6$$

$$10 + (-4) = 6$$

$$4 - 10 = -6$$

$$4 + (-10) = -6$$

$$-5 + 8 = 3$$

$$5 + (-8) = -3$$

$$5 - 8 = -3$$

$$8 - 5 = 3$$

$$8 + (-5) = 3$$

### Double Negatives

$$5 - (-8) = 5 + 8 = 13$$

$$-4 - (-10) = -4 + 10 = 6$$

$$12 - (-8) = 12 + 8 = 20$$

$$-16 - (-5) = -16 + 5 = -11$$



## Problem Set I: *Combining (Adding and Subtracting) Integers*

1	$-6 - 10$	2	$-8 - 10$	3	$5 - -16$	4	$5 - 12$	5	$-7 + 4$
6	$14 - 7$	7	$-11 + 12$	8	$-5 - 15$	9	$-7 + 1$	$\frac{1}{0}$	$-13 + 3$
$\frac{1}{1}$	$14 - 13$	$\frac{1}{2}$	$13 - 12$	$\frac{1}{3}$	$-6 - 15$	$\frac{1}{4}$	$14 - 8$	$\frac{1}{5}$	$5 - -6$
$\frac{1}{6}$	$-9 + 9$	$\frac{1}{7}$	$-4 - 9$	$\frac{1}{8}$	$-14 + 4$	$\frac{1}{9}$	$-16 - 12$	$\frac{2}{0}$	$15 - 4$
$\frac{2}{1}$	$2 - -9$	$\frac{2}{2}$	$12 - 6$	$\frac{2}{3}$	$-1 - 16$	$\frac{2}{4}$	$-1 - 10$	$\frac{2}{5}$	$7 - -9$
$\frac{2}{6}$	$13 - 9$	$\frac{2}{7}$	$-9 - 15$	$\frac{2}{8}$	$-14 - -10$	$\frac{2}{9}$	$-11 + 14$	$\frac{3}{0}$	$-3 - 1$
$\frac{3}{1}$	$10 - -7$	$\frac{3}{2}$	$-9 - 8$	$\frac{3}{3}$	$7 - 9$	$\frac{3}{4}$	$-1 - -13$	$\frac{3}{5}$	$7 - -4$
$\frac{3}{6}$	$-12 + 4$	$\frac{3}{7}$	$3 - 4$	$\frac{3}{8}$	$-11 - -1$	$\frac{3}{9}$	$-8 + 9$	$\frac{4}{0}$	$11 - 14$
$\frac{4}{1}$	$21 - 43$	$\frac{4}{2}$	$-19 - -10$	$\frac{4}{3}$	$-14 - 45$	$\frac{4}{4}$	$-1 - -20$	$\frac{4}{5}$	$11 - 4$
$\frac{4}{6}$	$-37 - 19$	$\frac{4}{7}$	$28 - 8$	$\frac{4}{8}$	$-13 - 44$	$\frac{4}{9}$	$14 - 49$	$\frac{5}{0}$	$14 - -6$
$\frac{5}{1}$	$19 - 38$	$\frac{5}{2}$	$18 - 29$	$\frac{5}{3}$	$-42 - 39$	$\frac{5}{4}$	$43 - 15$	$\frac{5}{5}$	$-4 - 21$
$\frac{5}{6}$	$-12 - 47$	$\frac{5}{7}$	$-24 - 35$	$\frac{5}{8}$	$-1 - -8$	$\frac{5}{9}$	$26 + 42$	$\frac{6}{0}$	$19 - 27$
$\frac{6}{1}$	$-45 - 14$	$\frac{6}{2}$	$-11 - 12$	$\frac{6}{3}$	$-1 - 41$	$\frac{6}{4}$	$26 - -32$	$\frac{6}{5}$	$40 - 30$
$\frac{6}{6}$	$41 - 22$	$\frac{6}{7}$	$11 - 17$	$\frac{6}{8}$	$44 - 24$	$\frac{6}{9}$	$41 - 47$	$\frac{7}{0}$	$12 - 21$
$\frac{7}{1}$	$-24 - 1$	$\frac{7}{2}$	$-13 - 38$	$\frac{7}{3}$	$16 - 38$	$\frac{7}{4}$	$16 + 38$	$\frac{7}{5}$	$-13 - 37$
$\frac{7}{6}$	$-5 - 60$	$\frac{7}{7}$	$70 - 15$	$\frac{7}{8}$	$70 - 90$	$\frac{7}{9}$	$-36 - 15$	$\frac{8}{0}$	$16 - 30$

## Combining (Adding and Subtracting) Fractions: Explanations and Examples

<p>Try to combine fractions using the method shown in these examples. If you are not comfortable with this method, work the problems using a method with which you are familiar.</p>	$\frac{2}{3} + \frac{3}{4}$ <p>The <i>least common denominator</i> (LCD) is the <i>least common multiple</i> of 3 and 4, which is 12.</p> $\frac{2}{3}\left(\frac{4}{4}\right) + \frac{3}{4}\left(\frac{3}{3}\right) = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$ <p>In algebra we prefer improper fractions to mixed numbers. Make sure they are in simplest form!</p>
$\frac{3}{8} + \frac{11}{12}$ <p>LCD = 24</p> $\frac{3}{8}\left(\frac{3}{3}\right) + \frac{11}{12}\left(\frac{2}{2}\right) = \frac{9}{24} + \frac{22}{24} = \frac{31}{24}$	$-\frac{3}{5} + \frac{1}{10}$ <p>LCD = 10</p> $-\frac{3}{5}\left(\frac{2}{2}\right) + \frac{1}{10}\left(\frac{1}{1}\right) = \frac{-6}{10} + \frac{1}{10} = \frac{-5}{10} = -\frac{1}{2}$
$-\frac{7}{16} - \frac{11}{24}$ <p>LCD = 48</p> $-\frac{7}{16}\left(\frac{3}{3}\right) - \frac{11}{24}\left(\frac{2}{2}\right) = \frac{-21}{48} - \frac{22}{48} = -\frac{43}{48}$	$\frac{2}{3} - \left(-\frac{3}{5}\right)$ <p>LCD = 15</p> $\frac{2}{3} - \left(-\frac{3}{5}\right) = \frac{2}{3} + \frac{3}{5} = \frac{2}{3}\left(\frac{5}{5}\right) + \frac{3}{5}\left(\frac{3}{3}\right)$ $= \frac{10}{15} + \frac{9}{15} = \frac{19}{15}$



## Problem Set II: *Combining (Adding and Subtracting) Fractions*

$\begin{array}{r} 8 \\ 1 \end{array}$	$-\frac{1}{4} - \frac{1}{3}$	$\begin{array}{r} 8 \\ 2 \end{array}$	$\frac{1}{5} + \frac{7}{5}$
$\begin{array}{r} 8 \\ 3 \end{array}$	$\frac{1}{4} - -\frac{3}{5}$	$\begin{array}{r} 8 \\ 4 \end{array}$	$\frac{4}{3} + \frac{5}{8}$
$\begin{array}{r} 8 \\ 5 \end{array}$	$\frac{2}{3} - \frac{4}{5}$	$\begin{array}{r} 8 \\ 6 \end{array}$	$-\frac{3}{2} + \frac{3}{2}$
$\begin{array}{r} 8 \\ 7 \end{array}$	$\frac{4}{5} + \frac{7}{8}$	$\begin{array}{r} 8 \\ 8 \end{array}$	$\frac{1}{2} - \frac{1}{3}$
$\begin{array}{r} 8 \\ 9 \end{array}$	$\frac{2}{3} - \left(-\frac{1}{2}\right)$	$\begin{array}{r} 9 \\ 0 \end{array}$	$-\frac{6}{7} - -\frac{4}{3}$
$\begin{array}{r} 9 \\ 1 \end{array}$	$-\frac{1}{3} + \frac{1}{5}$	$\begin{array}{r} 9 \\ 2 \end{array}$	$\frac{15}{8} - \frac{7}{8}$
$\begin{array}{r} 9 \\ 3 \end{array}$	$\frac{6}{5} + \frac{6}{7}$	$\begin{array}{r} 9 \\ 4 \end{array}$	$-\frac{9}{8} - \frac{3}{5}$

<b>9 5</b>	$\frac{11}{6} - \frac{3}{2}$	<b>9 6</b>	$\frac{7}{4} - \left(-\frac{6}{5}\right)$
<b>9 7</b>	$-\frac{7}{8} - \frac{1}{4}$	<b>9 8</b>	$\frac{1}{7} - \frac{3}{4}$
<b>9 9</b>	$-\frac{2}{5} + \frac{4}{3}$	<b>1 0 0</b>	$-\frac{1}{3} - -\frac{1}{3}$
<b>1 0 1</b>	$\frac{1}{3} - \frac{1}{2}$	<b>1 0 2</b>	$\frac{3}{7} + \frac{7}{4}$
<b>1 0 3</b>	$-\frac{4}{3} - \frac{11}{7}$	<b>1 0 4</b>	$\frac{3}{2} + \frac{4}{3}$
<b>1 0 5</b>	$\frac{1}{2} + \frac{1}{3} - \frac{1}{4}$	<b>1 0 6</b>	$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

## Multiplying and Dividing Fractions: Explanations and Examples

<p>Do you remember how to multiply fractions?</p> $\frac{2}{3} \times \frac{6}{7} = \frac{2 \times 6}{3 \times 7} = \frac{12}{21} = \frac{4}{7}$ <p>I strongly encourage you to SIMPLIFY BEFORE YOU MULTIPLY:</p> $\frac{2}{\cancel{3}^1} \times \frac{\cancel{6}^2}{7} = \frac{4}{7}$	<p><i>Examples—Multiplying Fractions</i></p> $\left(\frac{\cancel{3}^4}{\cancel{16}^4}\right) \left(\frac{\cancel{20}^5}{7}\right) = \frac{15}{28}$ $\left(\frac{\cancel{17}^1}{\cancel{6}^1}\right) \left(\frac{5}{13}\right) \left(\frac{\cancel{18}^3}{\cancel{17}^1}\right) = \frac{15}{13}$
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<p>Do you remember how to divide fractions?</p> <ol style="list-style-type: none"> <li>1. Leave the first fraction alone.</li> <li>2. Flip the second fraction (reciprocal)</li> <li>3. Multiply!</li> </ol> <p><i>Keep it, change it, flip it...</i></p>	<p><i>Examples—Dividing Fractions</i></p> $\frac{7}{12} \div \frac{5}{6} = \frac{7}{12} \times \frac{\cancel{6}^1}{5} = \frac{7}{10}$ $-\frac{3}{4} \div \frac{5}{7} = -\frac{3}{4} \times \frac{7}{5} = -\frac{21}{20}$
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### Problem Set III: Multiplying and Dividing Fractions

107	$-\frac{1}{2} \times \left(-\frac{1}{2}\right)$	108	$\frac{3}{7} * \left(-\frac{2}{5}\right)$
109	$\left(-\frac{3}{2}\right) \left(\frac{3}{2}\right)$	110	$\frac{2}{5} \times \left(-\frac{1}{3}\right)$
111	$\frac{8}{5} \times \left(-\frac{12}{7}\right)$	112	$-\frac{5}{6} * \frac{9}{13}$

113	$\frac{4}{7} \times \frac{3}{2}$	114	$\frac{1}{7} \left( \frac{1}{7} \right)$
115	$\frac{1}{2} \times \left( -\frac{14}{9} \right)$	116	$\frac{14}{9} * \frac{5}{3}$
117	$\frac{3}{5} \times \frac{5}{3}$	118	$-\frac{2}{3} \times \frac{5}{3}$
119	$\frac{3}{8} \left( -\frac{16}{9} \right)$	120	$\frac{5}{7} \times \frac{14}{15}$
121	$\left( \frac{18}{17} \right) \left( \frac{17}{12} \right)$	122	$\left( \frac{15}{7} \right) \left( \frac{11}{6} \right)$
123	$\frac{14}{15} \div \frac{11}{15}$	124	$\frac{2}{3} \div \frac{4}{7}$
125	$\frac{28}{23} \div \frac{5}{7}$	126	$-\frac{3}{8} \div \left( -\frac{3}{8} \right)$



127	$\frac{42}{19} \div \frac{14}{19}$	128	$\frac{2}{3}(24)$
129	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	130	$\left(\frac{17}{32}\right)\left(\frac{15}{19}\right)\left(\frac{19}{25}\right)\left(\frac{24}{17}\right)$
131	$\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) \div \frac{1}{2}$	132	$\left(\frac{x}{4}\right)\left(\frac{x}{4}\right)\left(\frac{x}{4}\right)$
133	$\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) =$	134	$\left(\frac{1}{2}\right)^3 =$
135	$\left(\frac{3}{8}\right)^2 =$	136	$\left(\frac{5}{4}\right)^3 =$
137	$\left(\frac{x}{4}\right)^3 =$ <i>Isn't this the same problem as #132?</i>	138	$\left(\frac{y}{2}\right)^5 =$

## Evaluating Expressions: Explanations and Examples

Remember your <b>order of operations</b> ?  <b>PEMDAS</b>  <b>P</b> arentheses <b>E</b> xponents <b>M</b> ultiply / <b>D</b> ivide <b>A</b> dd / <b>S</b> ubtract	Evaluate these expressions using these values for the variables:  $x = 3 \quad y = 4$	
	$ \begin{aligned} &2x + 5y \\ &= 2(3) + 5(4) \\ &= 6 + 20 \\ &= \mathbf{26} \end{aligned} $	$ \begin{aligned} &x^2 - y^2 \\ &= 3^2 - 4^2 \\ &= 9 - 16 \\ &= \mathbf{-7} \end{aligned} $
	$ \begin{aligned} &5x^2 + y^3 \\ &= 5(3)^2 + 4^3 \\ &= 5(9) + 64 \\ &= 45 + 64 \\ &= \mathbf{109} \end{aligned} $	$ \begin{aligned} &8(6x - 7y) \\ &= 8(6(3) - 7(4)) \\ &= 8(18 - 28) \\ &= 8(-10) \\ &= \mathbf{-80} \end{aligned} $



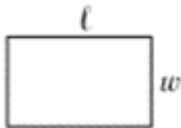

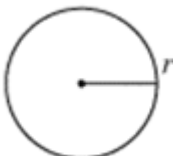
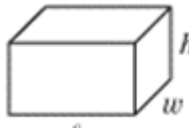
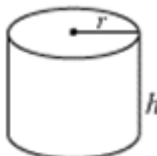
### Problem Set IV: Evaluating Expressions

Evaluate each expression if  $e = 2$ ,  $f = 3$ ,  $g = 4$ ,  $u = 0$ ,  $v = 5$ , and  $w = 1$ . Some of these problems are started for you.

1 3 9	$ \begin{aligned} &ev - f \\ &= (2)(5) - 3 \\ &= 10 - 3 \\ &= \end{aligned} $	1 4 0	$ \begin{aligned} &5g + 4w \end{aligned} $	1 4 1	$ \begin{aligned} &uv + fg \end{aligned} $
1 4 2	$ \begin{aligned} &w(v - f) + g \end{aligned} $	1 4 3	$ \begin{aligned} &(3g)(e + u) \\ &= 3(4)(2 + 0) \\ &= 3(4)(2) \\ &= \end{aligned} $	1 4 4	$ \begin{aligned} &w(v - u) + g \end{aligned} $
1 4 5	$ \begin{aligned} &(e + f)(g + v) \end{aligned} $	1 4 6	$ \begin{aligned} &e(u + v - w) \end{aligned} $	1 4 7	$ \begin{aligned} &(4e - 2f)(v + w) \end{aligned} $

## Using Perimeter, Area, and Volume Formulas: Explanations and Examples

Here are some common formulas for area and perimeter of common shapes. Examples are also given.

Rectangle	Triangle	Circle	Rectangular Prism	Cylinder
 $A = lw$	 $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$	 $A = \pi r^2$ $C = 2\pi r$	 $V = lwh$	 $V = \pi r^2 h$
Rectangle: $l = 5\text{ cm}$ $w = 3\text{ cm}$ $A = ?$  $A = lw$ $A = (5)(3)$ $A = \mathbf{15\text{ sq cm}}$	Triangle: $b = 18\text{ ft}$ $h = 7\text{ ft}$ $A = ?$  $A = \frac{1}{2}bh$ $A = \frac{1}{2}(18)(7)$  $A = \frac{1}{2} \cdot 126$ $A = 63$ $A = \mathbf{63\text{ sq ft}}$	Circle: $r = 8\text{ cm}$ $A = ?$ $C = ?$  $A = \pi r^2$ $A = \pi * 8^2$ $A = \mathbf{64\pi\text{ sq cm}}$ (This is called leaving your answer <i>in terms of</i> $\pi$ .)  $C = 2\pi r$ $C = 2\pi(8)$ $C = \mathbf{16\pi\text{ cm}}$ (Remember that <i>circumference</i> is a distance.)	Rectangular Prism $l = 5\text{ ft}$ $w = 8\text{ ft}$ $h = 3\text{ ft}$ $V = ?$  $V = lwh$ $V = (5)(8)(3)$ $V = \mathbf{120\text{ cu ft}}$	Cylinder $r = 4\text{ cm}$ $h = 10\text{ cm}$ $V = ?$  $V = \pi r^2 h$ $V = \pi * 4^2 * 10$ $= \pi * 16 * 10$ $= \mathbf{160\pi\text{ cu cm}}$



## Problem Set V: Using Perimeter, Area, and Volume Formulas

Find the indicated measurement. Write your answers in terms of  $\pi$  where possible.

1 4 8	Circle $r = 7\text{ ft}$ $A = ?$	1 4 9	Rectangular Prism $l = 2.5\text{ cm}$ $w = 4\text{ cm}$ $h = 12\text{ cm}$ $V = ?$	1 5 0	Rectangle $l = 4.1\text{ mm}$ $w = 3\text{ mm}$ $A = ?$
1 5 1	Triangle $b = 13\text{ ft}$ $h = 14\text{ ft}$ $A = ?$	1 5 2	Cylinder $r = 6\text{ ft}$ $h = 8\text{ ft}$ $V = ?$	1 5 3	Triangle $b = 5\text{ in}$ $h = 7\text{ in}$ $A = ?$
1 5 4	Here's the formula for the surface area of a cylinder: $SA_{\text{cylinder}} = 2\pi r^2 + 2\pi rh$ What is the surface area of this cylinder: $r = 4\text{ ft}$ $h = 7\text{ ft}$		1 5 5	Here's the formula for the surface area of a sphere: $SA_{\text{sphere}} = 4\pi r^2$ What is the surface area of this sphere: $r = 3\text{ m}$	

## ***“Percent” Means “Out of 100” : Explanations and Examples***

***Find the percent of a number***

<p>The “Of” Means “Multiply” Method</p> <p>What is 70% of 330?</p> $\begin{aligned} 70\% \text{ of } 330 \\ &= 0.7 \times 330 \\ &= 231 \end{aligned}$ <p>70% of 300 is <b>231</b>.</p>	<p>You can also set up and solve a proportion!</p> <p>What is 70% of 330?</p> <p>a number (<math>n</math>) out of 330 <math>\leftarrow \frac{n}{330} = \frac{70}{100} \rightarrow</math> 70% means 70 out of 100</p> $\begin{aligned} 100n &= (330)(70) \\ 100n &= 23100 \\ n &= 231 \end{aligned}$ <p>70% of 300 is <b>231</b>.</p>
<p>What is 35% of \$180 ?</p> $\begin{aligned} 35\% \text{ of } 180 \\ &= 0.35 \times 180 \\ &= 63 \end{aligned}$ <p>35% of \$180 is <b>\$63</b> .</p>	<p>What is 35% of \$180 ?</p> $\frac{n}{180} = \frac{35}{100}$ $\begin{aligned} 100n &= (180)(35) \\ 100n &= 6300 \\ n &= 63 \end{aligned}$ <p>35% of \$180 is <b>\$63</b> .</p>



### **Problem Set VI: “Percent” Means “out of 100”**

Answer these questions. Use the method of your choice, but try to use each method at least twice.

1 5 6	What is 45% of 150?	1 5 7	What is 37% of 37?
1 5 8	What is 8% of 150 ?	1 5 9	What is 13.5% of 10 ?
1 6 0	What is 200% of 39?	1 6 1	What is 40% of 80 ?

***Problem Set VI continues after this explanation of calculating percents mentally...  
A Lot of Percents Can Be Calculated Mentally! Explanations and Examples***

<b><i>Percent</i></b>	<b><i>Fraction Equivalent</i></b>	<b><i>What it Means</i></b>	<b><i>Example</i></b>
50%	$\frac{1}{2}$	$\div 2$	$50\% \text{ of } 36 = \frac{1}{2} \text{ of } 36 = 36 \div 2 = \mathbf{18}$
25%	$\frac{1}{4}$	$\div 4$	$25\% \text{ of } 36 = \frac{1}{4} \text{ of } 36 = 36 \div 4 = \mathbf{9}$
$33\frac{1}{3}\%$	$\frac{1}{3}$	$\div 3$	$33\frac{1}{3}\% \text{ of } 36 = \frac{1}{3} \text{ of } 36 = 36 \div 3 = \mathbf{12}$
10%	$\frac{1}{10}$	$\div 10$	$10\% \text{ of } 36 = \frac{1}{10} \text{ of } 36 = 36 \div 10 = \mathbf{3.6}$

<b><i>Percent</i></b>	<b><i>Fraction Equivalent</i></b>	<b><i>What it Means</i></b>	<b><i>Example</i></b>
75%	$\frac{3}{4}$	$\div 4 \times 3$	$75\% \text{ of } 60 = \frac{3}{4} \text{ of } 60 = 60 \div 4 \times 3 = \mathbf{45}$
$66\frac{2}{3}\%$	$\frac{2}{3}$	$\div 3 \times 2$	$66\frac{2}{3}\% \text{ of } 60 = \frac{2}{3} \text{ of } 60 = 60 \div 3 \times 2 = \mathbf{40}$
20%	$\frac{1}{5} \text{ or } \frac{2}{10}$	$\div 10 \times 2$	$20\% \text{ of } 60 = 60 \div 10 \times 2 = \mathbf{12}$
30%	$\frac{3}{10}$	$\div 10 \times 3$	$30\% \text{ of } 60 = 60 \div 10 \times 3 = \mathbf{18}$

***Some Cool Shortcuts...***

<b><i>Percent</i></b>	<b><i>Fraction Equivalent</i></b>	<b><i>What it Means</i></b>	<b><i>Cool Shortcut</i></b>	<b><i>Example</i></b>
25%	$\frac{1}{4}$	$\div 4$	$\div 2 \div 2$	$25\% \text{ of } \$80 = \frac{1}{4} \text{ of } \$80 = \$80 \div 2 \div 2 = \mathbf{\$20}$
$12\frac{1}{2}\%$	$\frac{1}{8}$	$\div 8$	$\div 2 \div 2 \div 2$	$12\frac{1}{2}\% \text{ of } \$80 = \frac{1}{8} \text{ of } \$80 = \$80 \div 2 \div 2 \div 2 = \mathbf{\$10}$

***A Few More...***

<b><i>Percent</i></b>	<b><i>What it Means</i></b>	<b><i>Example</i></b>
100%	$\times 1$	$100\% \text{ of } 49 = 1 \times 49 = \mathbf{49}$
200%	$\times 2$	$200\% \text{ of } 49 = 2 \times 49 = \mathbf{98}$
300%	$\times 3$	$300\% \text{ of } 49 = 3 \times 49 = \mathbf{147}$



## Problem Set VI (continued)

*A Lot of Percents Can Be Calculated Mentally!*

MENTAL MATH!

1 6 2	50% of \$30 = 25% of \$30 =	1 6 3	$33\frac{1}{3}\%$ of \$30 = $66\frac{2}{3}\%$ of \$30 =	1 6 4	10% of \$40 = 20% of \$40 = 30% of \$40 =
1 6 5	50% of 160 =	1 6 6	25% of 28 =	1 6 7	$12\frac{1}{2}\%$ of 32 =
1 6 8	200% of 17 =	1 6 9	75% of \$320	1 7 0	30% of \$320 =

Now try this! Look for patterns and use mental math!

1 7 1	<b>Percent of \$120</b>				1 7 2	<b>Percent of \$120</b>			
	50%		100%			10%		$12\frac{1}{2}\%$	
	25%		20%			25%		$33\frac{1}{3}\%$	
	$33\frac{1}{3}\%$		30%			75%		$66\frac{2}{3}\%$	
	10%		35%			100%		1%	
	5%		150%			125%		3%	
	15%		110%			110%		0.5%	

## 50 / 20 / 10 : What Every Algebra Student Should Know

### Explanations and Examples

**50:** Write all the factor pairs of the whole numbers from 1 to 50.

Here are the factors of 12: 1, 2, 3, 4, 6, 12

Since factors are multiplied to get a product ( $factor \times factor = product$ ), the factors of 12 can be written in pairs:

$$1 \times 12$$

$$2 \times 6$$

$$3 \times 4$$

Or, more efficiently, they can be written in a table like this:

12	
1	12
2	6
3	4

**Other Numbers:**

30	
1	30
2	15
3	10
5	6

7	
1	7

*Oh yeah—7 is a prime number!*

25	
1	25
5	5

**20:** Write (and ideally memorize) the first twenty square numbers.

$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	...
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**10:** Write (and ideally memorize) the first twenty cube numbers.

$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	...
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# 50 / 20 / 10 What Every Algebra Student Should Know

**Factor Pairs:** Write the positive factors for each number. Write them in pairs, as demonstrated for the numbers 12 and 13.

<b>1</b>		<b>2</b>		<b>3</b>		<b>4</b>		<b>5</b>		<b>6</b>	
<b>7</b>		<b>8</b>		<b>9</b>		<b>10</b>		<b>11</b>		<b>12</b>	
										1 2 3	12 6 4
<b>13</b>		<b>14</b>		<b>15</b>		<b>16</b>		<b>17</b>		<b>18</b>	
1	13										
<b>19</b>		<b>20</b>		<b>21</b>		<b>22</b>		<b>23</b>		<b>24</b>	
<b>25</b>		<b>26</b>		<b>27</b>		<b>28</b>		<b>29</b>		<b>30</b>	
<b>31</b>		<b>32</b>		<b>33</b>		<b>34</b>		<b>35</b>		<b>36</b>	
<b>37</b>		<b>38</b>		<b>39</b>		<b>40</b>		<b>41</b>		<b>42</b>	
<b>43</b>		<b>44</b>		<b>45</b>		<b>46</b>		<b>47</b>		<b>48</b>	
<b>49</b>						<b>50</b>					

<b>Squares</b>
$1^2 =$
$2^2 =$
$3^2 =$
$4^2 =$
$5^2 =$
$6^2 =$
$7^2 =$
$8^2 =$
$9^2 = 81$
$10^2 =$
$11^2 =$
$12^2 =$
$13^2 =$
$14^2 =$
$15^2 =$
$16^2 =$
$17^2 =$
$18^2 =$
$19^2 =$
$20^2 =$

<b>CUBES</b>	$1^3 =$	$2^3 =$	$3^3 =$	$4^3 =$	$5^3 =$	$6^3 =$	$7^3 =$	$8^3 =$	$9^3 =$	$10^3 =$
					125					



## Problem Set VII (optional): *Some Problems You May Enjoy...*

These are not required! It's perfectly OK if you don't understand all of these!

Calculator use is OK, but you may find you don't need one!

173	If 6 and $x$ have the same mean as the set $\{2, 4, 24\}$ , what is the value of $x$ ?
174	If $a \bowtie b = b^a * a^{b-a}$ , what is the value of $(2 \bowtie 5)$ ?
175	If $f(x) = x^2 - 2x - 5$ and $g(x) = x^3 - 11$ , what is the value of $f(4) + g(5)$ ?
176	The prime factorization of 720 is $2^a * 3^b * 5^c$ . What is the value of $a + b + c$ ?
177	What digit is in the hundred-thousandths place when $(0.5)^5$ is simplified and written as a decimal?
178	If $16 \times 2^x = 2^y + 2^y + 2^y + 2^y$ , what is the positive difference between $x$ and $y$ ?
179	If 30% of $x$ is 12,345, what is 40% of $x$ ?
180	A rectangle with area of 72 square units has vertices $(2, 3)$ , $(2n + 2, 3)$ , $(2n + 2, n + 3)$ and $(2, n + 3)$ with $n > 0$ . What is the value of $n$ ?
181	John has a total of \$1.40 in nickels and dimes, with twice as many nickels as there are dimes. How many nickels does John have?

## ANSWERS...

### Problem Set I: *Combining (Adding and Subtracting) Integers*

1	-16	2	-18	3	21	4	-7	5	-3	6	7	7	1	8	-20	9	-6	10	-10
11	1	12	1	13	-21	14	6	15	11	16	0	17	-13	18	-10	19	-28	20	11
21	11	22	6	23	-17	24	-11	25	16	26	4	27	-24	28	-4	29	3	30	-4
31	17	32	-17	33	-2	34	12	35	11	36	-8	37	-1	38	-10	39	1	40	-3
41	-22	42	-9	43	-59	44	19	45	7	46	-56	47	20	48	-57	49	-35	50	20
51	-19	52	-11	53	-81	54	28	55	-25	56	-59	57	-59	58	7	59	68	60	-8
61	-59	62	-23	63	-42	64	58	65	10	66	19	67	-6	68	20	69	-6	70	-9
71	-25	72	-51	73	-22	74	54	75	-50	76	-65	77	55	78	-20	79	-51	80	-14

### Problem Set II: *Combining (Adding and Subtracting) Fractions*

81	$-\frac{7}{12}$	82	$\frac{8}{5}$	83	$\frac{17}{20}$	84	$\frac{47}{24}$	85	$-\frac{2}{15}$	86	0	87	$\frac{67}{40}$	88	$\frac{1}{6}$	89	$\frac{7}{6}$	90	$\frac{10}{21}$
91	$-\frac{2}{15}$	92	1	93	$\frac{72}{35}$	94	$-\frac{69}{40}$	95	$\frac{1}{3}$	96	$\frac{59}{20}$	97	$-\frac{9}{8}$	98	$-\frac{17}{28}$	99	$\frac{14}{15}$	100	0
101	$-\frac{1}{6}$	102	$\frac{61}{28}$	103	$-\frac{61}{21}$	104	$\frac{17}{6}$	105	$\frac{7}{12}$	106	$\frac{15}{16}$								

### Problem Set III: *Multiplying and Dividing Fractions*

$\frac{1}{07}$	$\frac{1}{4}$	$\frac{1}{08}$	$-\frac{6}{35}$	$\frac{1}{09}$	$-\frac{9}{4}$	$\frac{1}{10}$	$-\frac{2}{15}$	$\frac{1}{11}$	$-\frac{96}{35}$	$\frac{1}{12}$	$-\frac{15}{26}$	$\frac{1}{13}$	$\frac{6}{7}$	$\frac{1}{14}$	$\frac{1}{49}$	$\frac{1}{15}$	$-\frac{7}{9}$	$\frac{1}{16}$	$\frac{70}{27}$
$\frac{1}{17}$	1	$\frac{1}{18}$	$-\frac{10}{9}$	$\frac{1}{19}$	$-\frac{2}{3}$	$\frac{1}{20}$	$\frac{2}{3}$	$\frac{1}{21}$	$\frac{3}{2}$	$\frac{1}{22}$	$\frac{55}{14}$	$\frac{1}{23}$	$\frac{14}{11}$	$\frac{1}{24}$	$\frac{7}{6}$	$\frac{1}{25}$	$\frac{196}{115}$	$\frac{1}{26}$	1
$\frac{1}{27}$	3	$\frac{1}{28}$	16	$\frac{1}{29}$	$\frac{1}{16}$	$\frac{1}{30}$	$\frac{9}{20}$	$\frac{1}{31}$	1	$\frac{1}{32}$	$\frac{x^3}{64}$	$\frac{1}{33}$	$\frac{16}{81}$	$\frac{1}{34}$	$\frac{1}{8}$	$\frac{1}{35}$	$\frac{9}{64}$	$\frac{1}{36}$	$\frac{125}{64}$
$\frac{1}{37}$	$\frac{x^3}{64}$	$\frac{1}{38}$	$\frac{y^5}{32}$																

### Problem Set IV: *Evaluating Expressions*

$\frac{1}{39}$	7	$\frac{1}{40}$	24	$\frac{1}{41}$	12	$\frac{1}{42}$	6	$\frac{1}{43}$	24	$\frac{1}{44}$	9	$\frac{1}{45}$	45	$\frac{1}{46}$	8	$\frac{1}{47}$	12
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### Problem Set V: *Using Perimeter, Area, and Volume Formulas*

$\frac{1}{48}$	$49\pi$ sq ft	$\frac{1}{49}$	120 cubic cm	$\frac{1}{50}$	$12.3$ sq mm	$\frac{1}{51}$	$91$ sq ft
$\frac{1}{52}$	$288\pi$ cu ft	$\frac{1}{53}$	$17\frac{1}{2}$ sq in	$\frac{1}{54}$	$88\pi$ sq ft	$\frac{1}{55}$	$36\pi$ sq m

### Problem Set VI: *“Percent” Means “Out of 100”*

$\frac{1}{56}$	67.5	$\frac{1}{57}$	13.69	$\frac{1}{58}$	12	$\frac{1}{59}$	1.35	$\frac{1}{60}$	78	$\frac{1}{61}$	32
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$\frac{1}{62}$	\$15 \$7.50	$\frac{1}{63}$	\$10 \$20	$\frac{1}{64}$	\$4 \$8 \$12	$\frac{1}{65}$	80	$\frac{1}{66}$	7	$\frac{1}{67}$	4	$\frac{1}{68}$	34	$\frac{1}{69}$	\$240	$\frac{1}{70}$	\$96
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$\frac{1}{71}$	<b>Percent of \$120</b>				<b>Percent of \$120</b>			
	50%	\$60	100%	\$120	10%	\$12	$12\frac{1}{2}\%$	\$15
	25%	30	20%	24	25%	30	$33\frac{1}{3}\%$	40
	$33\frac{1}{3}\%$	40	30%	36	75%	90	$66\frac{2}{3}\%$	80
	10%	12	35%	42	100%	120	1%	1.20
	5%	6	150%	180	125%	150	3%	3.60
	15%	18	110%	132	110%	132	0.5%	0.60

### Problem Set VII (optional): *Some Problems You May Enjoy...*

$\frac{1}{73}$	14	$\frac{1}{74}$	200	$\frac{1}{75}$	117	$\frac{1}{76}$	7	$\frac{1}{77}$	5	$\frac{1}{78}$	2	$\frac{1}{79}$	16,460	$\frac{1}{80}$	6	$\frac{1}{81}$	14
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